## 1. Trigonometry reminder

**E.3792** Let $\alpha$ be a real number. Simplify the following entries:

(a) $\cos\left(\frac{\pi}{2}+\alpha\right)$  (b) $\sin\left(\alpha+3\cdot\pi\right)$

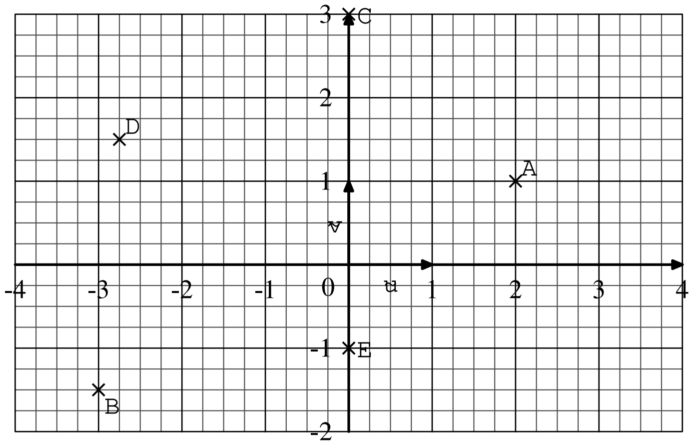
(c) $\cos\left(\alpha-\frac{\pi}{2}\right)$  (d) $\sin\left(\frac{\pi}{2}-\alpha\right)$

**E.3793** Solve the following equations:

(a) $\sin\left(x+\frac{\pi}{4}\right)=\frac{\sqrt{3}}{2}$  (b) $\cos\left(2x+\frac{\pi}{3}\right)=\frac{\sqrt{2}}{2}$

## 2. Graphical representation and algebraic writing

**E.3784** Consider the complex plane with a reference frame $\left(O\,;\overrightarrow{u}\,;\overrightarrow{v}\right)$ orthonormal direct and the five points shown below:




1. Determine the algebraic writings of the affixes of the points $A$, $B$, $C$, $D$, $E$.
2. Place in the plane the points $F$, $G$, $H$ and $I$ with respective affixes $z_1$, $z_2$, $z_3$ and $z_4$ defined by:

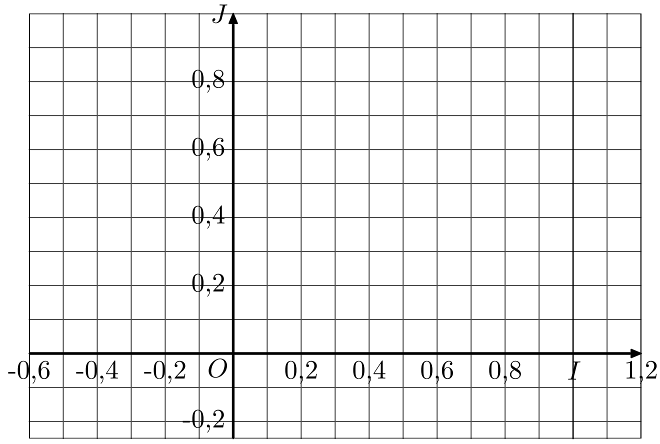
$$z_1=3-\text{i} \quad ; \quad z_2=\frac{3}{2}\cdot\text{i}$$

$$z_3=-\frac{7}{4}+2\cdot\text{i} \quad ; \quad z_4=\frac{3}{2}+\frac{9}{4}\cdot\text{i}$$

3. Determine the affix of the middle of segment $[EF]$.

**E.6792** We equip the complex plane with an orthonormal direct reference frame $\left(O\,;\overrightarrow{u}\,;\overrightarrow{v}\right)$. We define the sequence $(z_n)$ of complex numbers by: $z_0=1$ ; $z_{n+1}=\left(\frac{3}{4}+\frac{\sqrt{3}}{4}\cdot\text{i}\right)\cdot z_n$

1. Determine the algebraic expression of the first four terms of the sequence $(z_n)$.
2. Using your calculator, give the algebraic expression of the terms $z_4$ and $z_5$, rounding the real and imaginary parts to the nearest $10^{-2}$.
3. For any natural number $n$, let $M_n$ be the images of the complex number $z_n$.
   (a) Place the points $M_0,\ldots,M_5$ on the coordinate plane below:




   (b) What conjecture can be made about the nature of triangles $OM_0M_1$, $OM_1M_2$, and $OM_2M_3$?

## 3. Modulus of a complex number

**E.5345** Determine the modulus of the following complex numbers:

(a) $1-2\cdot\text{i}$  (b) $-5\cdot\text{i}$  (c) $(3-2\cdot\text{i})(2+\text{i})$

**E.8568** Determine the modulus of the following complex numbers:

(a) $-\text{i}\cdot(1-2\cdot\text{i})$  (b) $\frac{1+\text{i}\cdot\sqrt{3}}{3\cdot\text{i}}$  (c) $\frac{3-\text{i}\cdot\sqrt{3}}{-\sqrt{3}+\text{i}}$

## 4. Geometry and module

**E.3843** In the complex plane with a reference frame $(O\,;\,\overrightarrow{u}\,;\,\overrightarrow{v})$, consider the points $A$, $B$, $C$ with respective affixes:

$$z_A=1+3\cdot\text{i} \quad ; \quad z_B=-\sqrt{2}+2\cdot\text{i} \quad ; \quad z_C=\frac{\sqrt{2}}{2}+\text{i}\cdot\Big(\frac{\sqrt{6}}{2}+2\Big)$$

Let $I$ be the point of the plane with affix $z_I=2\cdot\text{i}$

Show that the points $A$, $B$, $C$ belong to the same circle with center $I$. Specify the radius of this circle.

## 5. Relationship $\left|z\right|^2=z\cdot\overline{z}$

**E.8569** For any complex number $z$, we define the complex number $z'$ by:

$$z'=\frac{(3+4\cdot\text{i})\cdot z+5\cdot\overline{z}}{6}$$

1. For any complex number $z$, establish the equality:

$$\frac{z'-z}{1+2\cdot\text{i}}=\frac{z+\overline{z}}{6}+\text{i}\cdot\frac{z-\overline{z}}{3}$$

2. Deduce that the number $\dfrac{z'-z}{1+2\cdot\text{i}}$ is a real number.

**E.8570** Establish the equations below:

a. $\left|\dfrac{5\cdot\text{i}}{5-5\cdot\text{i}}\right|=\dfrac{\sqrt{2}}{2}$ b. $\left|\dfrac{1+2\cdot\text{i}}{5+5\cdot\text{i}}\right|=\dfrac{\sqrt{10}}{10}$

## 6. Module: algebraic properties

**E.3846** Establish the following equality for any non-zero complex number $z$: $\Big(\dfrac{1}{z^2}-1\Big)\overline{\Big(z^2-1\Big)}=-\overline{z}^2\cdot\left|\dfrac{1}{z^2}-1\right|^2$

## 7. Modules and Cartesian equations

**E.3844** Consider the complex plane provided with a reference frame $(O\,;\,\overrightarrow{u}\,;\,\overrightarrow{v})$ orthonormal direct:

1. Let $z$ be a complex number solution of the equation $(E)$:
   $(E):\ |z-2+\text{i}|=5$
   Note $M$ the image of the complex number $z$
   a. Translate equation $(E)$ in terms of distance.
   b. Justify that the set of points $M$ of affix $z$ verifying the relation $(E)$ is a circle. Specify its center and radius.
   c. Let $a+\text{i}\cdot b$ be the algebraic writing of the affix of the point $M$; determine a relation on $a$ and $b$ characterizing the relation $(E)$.
2. Let $z$ be a complex number solution of the equation $(F)$:
   $(F):\ |z+\text{i}|=|z+1-2\cdot\text{i}|$
   We note $M$ the image of the complex number $z$
   a. Translate equation $(F)$ in terms of distance.
   b. Justify that the set of points $M$ of affix $z$ verifying the relation $(F)$ is a straight line. Specify its nature.
   c. Let $a+\text{i}\cdot b$ be the algebraic writing of the affix of the point $M$; determine a relation on $a$ and $b$ characterizing the relation $(E)$.

**E.3813** ⚠ The plane is referred to the $(O\,;\,\overrightarrow{u}\,;\,\overrightarrow{v})$ direct orthornormal reference frame. Consider the points $A$ and $B$ of affixes:

$$z_A=-1+\text{i}\cdot\sqrt{3} \quad ; \quad z_B=-1-\text{i}\cdot\sqrt{3}$$

1. Establish that the set $\Gamma_2$ of points $M$ of affix $z$ that verify:
   $2(z+\overline{z})+z\cdot\overline{z}=0$
   is a circle with center $\Omega$ and affix $-2$. Specify its radius. Construct $\Gamma_2$.
2. Check that points $A$ and $B$ are $\Gamma_2$ elements

## 8. Set $\mathbb{U}$

**E.8573**

> **Definition:** note $\mathbb{U}$ the set of complex numbers $z$ verifying $|z|=1$.
>
> Note: $\mathbb{U}=\{z\in\mathbb{C}\ |\ |z|=1\}$ ; $\mathbb{U}\subset\mathbb{C}$

1. Justify that the complex numbers below belong to $\mathbb{U}$:
   a. $z_1=-\text{i}$ b. $z_2=\dfrac{\sqrt{3}}{2}-\dfrac{1}{2}\cdot\text{i}$ c. $z_3=-\dfrac{\sqrt{2}}{2}+\dfrac{\sqrt{2}}{2}\cdot i$
2. Let $z$ and $z'$ be two complex numbers belonging to $\mathbb{U}$, show that the product $z\cdot z'$ belongs to $\mathbb{U}$.
3. Let $z$ be an element of $\mathbb{U}$, show that the three expressions below define a complex number belonging to $\mathbb{U}$:

ⓐ $-z$ ⓑ $\frac{1}{z}$ ⓒ $z^2$

④ Show that, for any complex number $z$ $(z\in\mathbb{C})$, the complex number $\frac{z}{|z|}$ belongs to $\mathbb{U}$.

**E.8574** Establish that the complex numbers below belong to $\mathbb{U}$:

ⓐ $z_1 = \frac{1-7{\cdot}\mathrm{i}}{-5+5{\cdot}\mathrm{i}}$ ⓑ $z_2 = \frac{2-9{\cdot}\mathrm{i}}{-6+7{\cdot}\mathrm{i}}$ ⓒ $z_3 = \frac{4+7{\cdot}\mathrm{i}}{8-\mathrm{i}}$

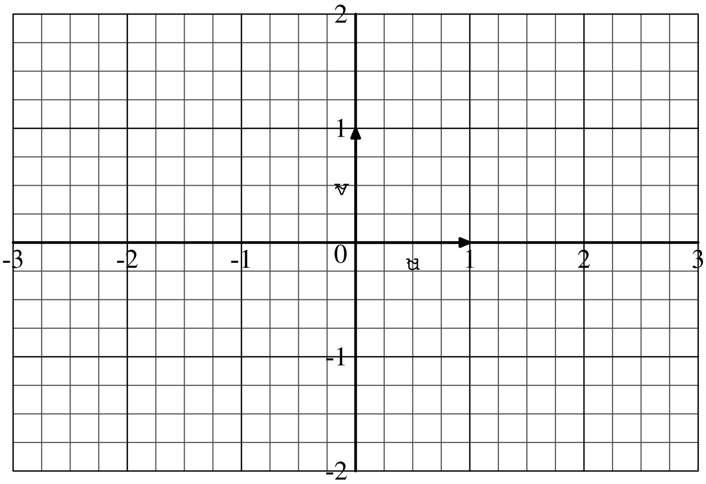
## 9. *Argument on the whole* $\mathbb{U}$

**E.8575**

**definition:** Consider the complex plane provided with a reference frame $\left(O\,;\overrightarrow{u}\,;\overrightarrow{v}\right)$ orthonormal direct.
For any complex number $z$ belonging to $\mathbb{U}$, we call **argument of $z$** the measure of the oriented angle $\left(\overrightarrow{u}\,;\overrightarrow{OM}\right)$ where $M$ is the image of the point $z$.

① In the representation below of the complex plane provided with a reference frame $\left(O\,;\overrightarrow{u}\,;\overrightarrow{v}\right)$ orthonormal direct, place the points $M_1$, $M_2$, $M_3$ belonging to the trigonometric circle and whose respective arguments of the affixes of these points have values:

ⓐ $\theta_1 = \frac{\pi}{4}$ ⓑ $\theta_2 = \frac{5\pi}{6}$ ⓒ $\theta_3 - \frac{2\pi}{3}$




**Proposition:** in the complex plane provided with a reference frame $\left(O\,;\overrightarrow{u}\,;\overrightarrow{v}\right)$ orthonormal direct. The point $M$ belonging to the trigonometric circle whose affix $z$ has argument $\theta$ admits as algebraic writing:
$z = \cos\theta + \mathrm{i}{\cdot}\sin\theta$

② Give the algebraic writings of the affixes of the points $M_1$, $M_2$, $M_3$ defined in question ①.

## 10. *Remarkable arguments and angles*

**E.5565**

**Definition:** consider the complex plane provided with a reference frame $\left(O\,;\overrightarrow{u}\,;\overrightarrow{v}\right)$ orthonormal direct.
Let $z$ be a non-zero complex number, we call **argument of** $z$ the value of the argument of the complex number $\frac{z}{|z|}$

Determine the arguments of the complex numbers below:

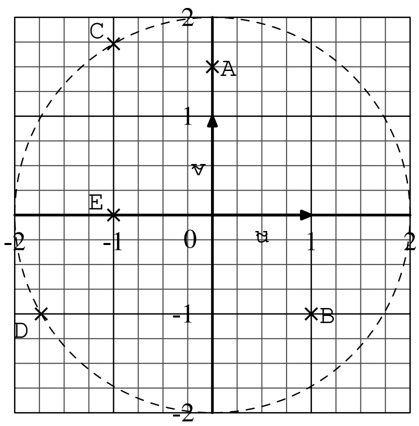
ⓐ $z_1 = 1+\mathrm{i}$ ⓑ $z_2 = -3+3{\cdot}\sqrt{3}{\cdot}\mathrm{i}$ ⓒ $z_3 = \sqrt{3}-\mathrm{i}$

**E.3795**

**Geometric interpretation:** in the plane provided with a reference frame $\left(O\,;\overrightarrow{u}\,;\overrightarrow{v}\right)$ orthonormal direct, for any point $M$ different from $O$. The argument of the affix of the point $M$ has as its value the measure of the angle oriented $\left(\overrightarrow{u}\,;\overrightarrow{OM}\right)$.

Consider the plane provided with a reference frame $\left(O\,;\overrightarrow{u}\,;\overrightarrow{v}\right)$ orthogonal direct represented below:

The circle $\mathscr{C}$ with center $O$ and radius 2 is shown dotted; points $C$ and $D$ belong to the circle $\mathscr{C}$.




① Determine the moduli and arguments of the affixes of the points $A$, $B$, $C$, $D$ and $E$.

② Place the points $F$ and $G$ of affixes $z$ and $z'$ respectively, verifying :

$$\begin{cases} |z| = \sqrt{2} \\ \arg(z) = -\dfrac{3\pi}{4} \end{cases} \quad ; \quad \begin{cases} |z'| = 2 \\ \arg(z') = -\dfrac{\pi}{2} \end{cases}$$

## 11. *Argument and approximate value*

E.8572 For each of the complex numbers below, determine the approximate value of their argument to the nearest hundredth of a radian.

(a) $z_1 = 2+\mathrm{i}$ (b) $z_2 = -3-2{\cdot}\mathrm{i}$ (c) $z_3 = 1-2{\cdot}\mathrm{i}$

E.5346 Consider the complex numbers below :

$$z_1 = 1+2{\cdot}\mathrm{i} \quad ; \quad z_2 = 5-2{\cdot}\mathrm{i} \quad ; \quad z_3 = -1-2{\cdot}\mathrm{i}$$

1. Determine the modulus of each of the above complex numbers.
2. Determine, to the nearest hundredth of a radian, the value of the argument of each of the above complex numbers.

## 12. *Trigonometric writing*

E.5566 Determine the algebraic writing of the complex numbers $z_5$ and $z_6$ non-zero defined by :

(a) $|z_5| = 5 \quad ; \quad \arg(z_5) = -\dfrac{\pi}{3}$

(b) $|z_6| = 2 \quad ; \quad \arg(z_6) = -\dfrac{\pi}{2}$

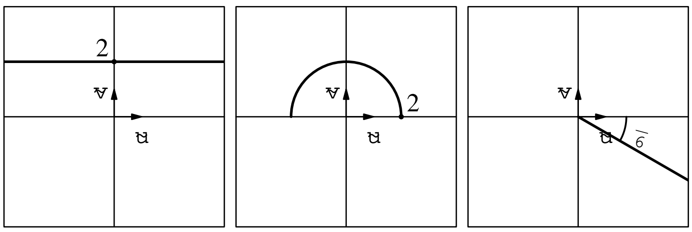
E.3797 Determine the trigonometric form of each of the complex numbers below :

(a) $z_1 = -\dfrac{1}{2}+\mathrm{i}{\cdot}\dfrac{\sqrt{3}}{2}$ (b) $z_2 = \sqrt{3}+\mathrm{i}$

(c) $z_3 = -1-\mathrm{i}$ (d) $z_4 = \dfrac{\sqrt{2}}{2}+\mathrm{i}{\cdot}\dfrac{\sqrt{2}}{2}$

## 13. *Moduli, arguments and geometric loci*

E.5380 Consider the plane provided with a reference frame $(O\,;\overrightarrow{u}\,;\overrightarrow{v})$ orthonormal and direct.
For each of the parts of the plane shown below, describe this subset of $\mathbb{C}$ using the algebraic writing, modulus, argument of the affixes of their points.




E.3796 Consider the plane provided with a reference frame $(O\,;\overrightarrow{u}\,;\overrightarrow{v})$ orthonormal direct. Describe the set of points $M$ of the plane whose affix $z$ verifies the following conditions :

(a) $|z| = 1$ (b) $|z| \leqslant 3$ (c) $1 \leqslant |z| \leqslant 2$

(d) $\arg(z) = \dfrac{\pi}{2}$ (e) $\big|\arg(z)\big| = \dfrac{\pi}{4}$

## 14. *Geometric locations*

E.6806 Consider the three sets, defined below, of pairs of reals :

- $\mathcal{E}_1 = \big\{(x\,;y) \mid 2{\cdot}x+y-1=0\big\}$
- $\mathcal{E}_2 = \big\{(x\,;y) \mid x^2+y^2-2{\cdot}x+4{\cdot}y-4=0\big\}$
- $\mathcal{E}_3 = \big\{(x\,;y) \mid x{\cdot}y+2{\cdot}y=0\big\}$

Consider the plane provided with a $(O\,;I\,;J)$ direct orthonormal coordinate system. Give for each of the three sets above :

- the nature of their representation in the plane,
- elements characterizing their representation.

E.6776 To any complex number $z$, we associate a complex number $z'$ defined by :

$$z' = z^2 + 4{\cdot}z + 3$$

Determine the set $\mathcal{E}$ of complex numbers $z$ such that $z'$ is a real number.

E.6782 Note $\mathbb{C}$ the set of complex numbers. The complex plane is provided with a reference frame $(O\,;\,\overrightarrow{u}\,;\,\overrightarrow{v})$ orthonormal direct.

Consider the function $f$ which associates: $f(z)=z^2+2{\cdot}z+9$

Let $z$ be a complex number, such that $z=x+\mathrm{i}{\cdot}y$ where $x$ and $y$ are real numbers.

1. Show that the algebraic form of $f(z)$ is:
   $f(z)=\big(x^2-y^2+2{\cdot}x+9\big)+\mathrm{i}{\cdot}\big(2{\cdot}x{\cdot}y+2{\cdot}y\big)$
2. Note $(\mathcal{E})$ the set of points in the complex plane whose affix $z$ is such that $f(z)$ is a real number.
   Show that $(\mathcal{E})$ is the union of two straight lines $D_1$ and $D_2$ whose equations will be specified.

E.6353 *Of the answers given, only one is correct. Which one? Justify your answer.*

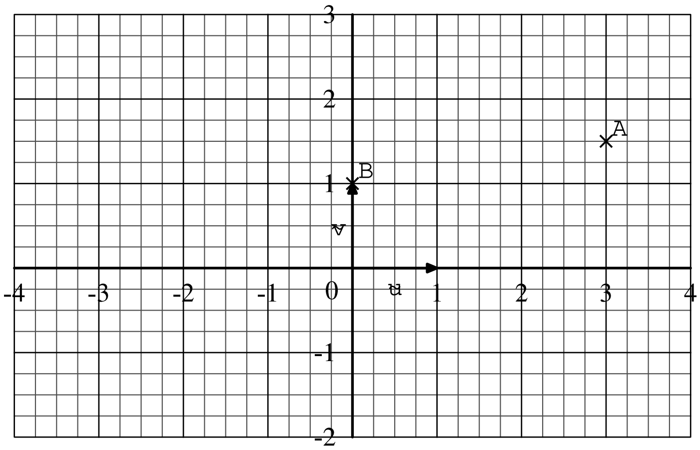
Consider the set $\mathcal{E}$ of complex numbers $z$ verifying:
$\dfrac{1}{z^2+1}$ be a real.

The set $\mathcal{E}$ is:

1. the set of real numbers;
2. the set of pure imaginary numbers deprived of i and $-\mathrm{i}$;
3. the union of the set of real numbers and the set of pure imaginary numbers deprived of i and $-\mathrm{i}$;
4. the number 0.

# 15. *Plan transformation*

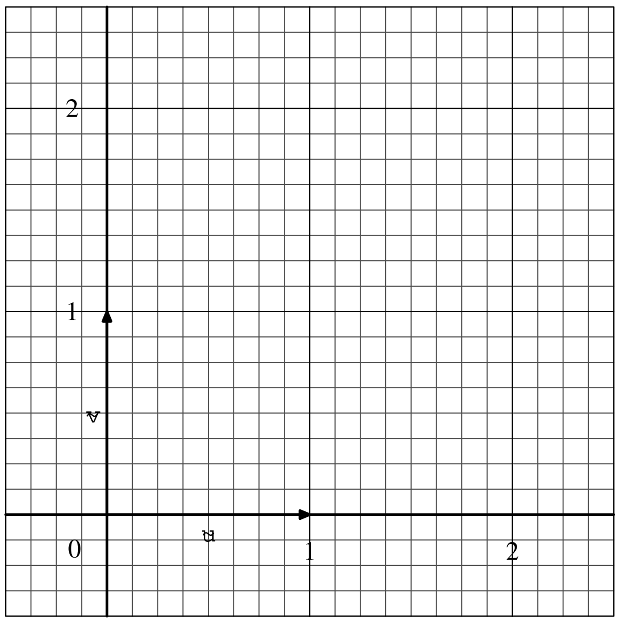
E.3785 Consider the coplex plane provided with a $(O\,;\,\overrightarrow{u}\,;\,\overrightarrow{v})$ orthonormal direct and the points $A$ and $B$ shown below:




Let $z_A$ and $z_B$ be the respective affixes of the points $A$ and $B$.

1. Give the algebraic writings of the affixes of the points $A$, $B$.
2. (a) Place the point $C$ of affix $-z_A$.
   (b) Which geometric transformation transforms the point $A$ into $C$?
3. (a) Place the point $D$ of affix $\overline{z_A}$.
   (b) What geometric transformation transforms the point $A$ into $D$?
4. (a) Place the point $E$ of affix $-\overline{z_A}$.
   (b) Which geometric transformation transforms the point $A$ into $E$?
5. (a) Place the point $F$ of affix $\dfrac{1}{2}{\cdot}z_A$.
   (b) What can be said about the position of the point $F$?
6. (a) Place the point $G$ of affix $z_A+\big(-2-2{\cdot}\mathrm{i}\big)$.
   (b) Which geometric transformation transforms the point $A$ into $G$?
7. Consider the point $H$ of affix $z_H$ verifying equality:
   $z_H-z_B=\dfrac{1}{2}{\cdot}(z_A-z_B)$
   (a) By solving the equation, determine the algebraic writing of the affix $z_H$ of the point $H$.
   (b) What can be the position of the point $H$?

E.6796 Consider the complex plane provided with a $(O\,;\,\overrightarrow{u}\,;\,\overrightarrow{v})$ orthonormal direct reference frame.




1. Consider the points $A$, $B$, $C$ with respective affixes:
   $z_1=0.5+0.5{\cdot}\mathrm{i}\quad;\quad z_2=1+\mathrm{i}\quad;\quad z_3=2+2{\cdot}\mathrm{i}$
   Place the points $A$, $B$ and $C$.
2. Consider the function $f$ defined on the set of non-zero complex numbers by:
   $f(z)=-\dfrac{1}{z^2}$
   (a) Determine the complex numbers $z'_1$, $z'_2$, $z'_3$ respective images by the function $f$ of $z_1$, $z_2$, $z_3$.
   (b) Place the points $A'$, $B'$, $C'$ respective images of the complex numbers $z'_1$, $z'_2$, $z'_3$.
3. Can we say that the transformation of the plane obtained using the complex function $f$ is an isometry? Justify your answer.

# 16. *Plane transformation and geometric locus*

**E.6020** To any complex number $z$ such as $z \neq 2$, we associate the complex number $z'$ defined by: $z' = \dfrac{z+3}{z-1}$

1. We note $x+\mathrm{i}\cdot y$, where $x\in\mathbb{R}$ and $y\in\mathbb{R}$, the algebraic writing of the complex number $z$.
   Give the algebraic writing of the number $z'$, associated with $z$, as a function of $x$ and $y$.

2. In the complex plane with a reference frame $\left(O\,;\,\overrightarrow{u}\,;\,\overrightarrow{v}\right)$ orthonormal direct:

   a. Consider the Cartesian equation:
      $(E):\ x^2+y^2+2x-3=0$
      Justify that, in the plane, the set of solutions of $(E)$ is the circle $\mathscr{C}$ of center $(-1\,;0)$ and radius 2.

   b. Determine the set of complex numbers $z$ such that the associated complex number $z'$ is a pure imaginary.

**E.6797** Consider the function $f$ defined on $\mathbb{C}$ by the relation:
$f(z) = z + \overline{z}^2$

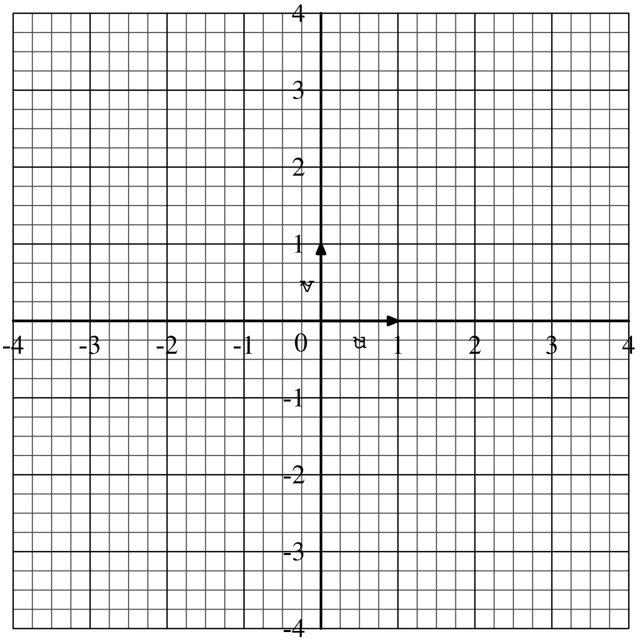
1. Let's note $a$ and $b$ the real numbers such that $z$ admit as algebraic writing $z = a+\mathrm{i}\cdot b$.

   Give the algebraic writing of the complex number $f(z)$.

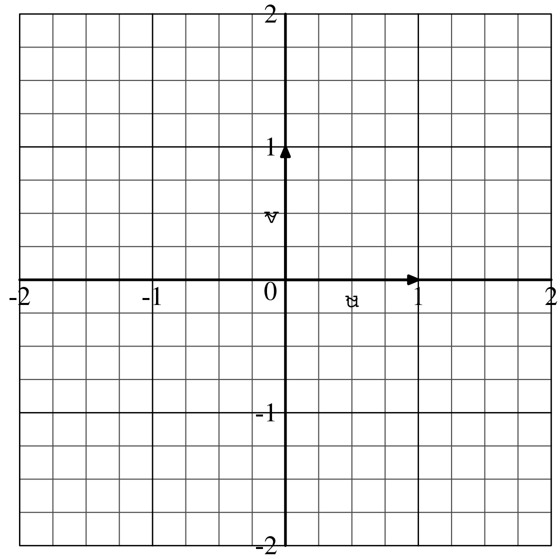
2. In this question, we seek to determine the set $\mathcal{E}$ of complex numbers $z$ such that $f(z)$ is a real. That is:
   $\mathcal{E} = \left\{z\in\mathbb{C} \mid Im\left[f(z)\right]=0\right\}$

   a. Let $z$ be a complex number belonging to $\mathcal{E}$. Determine the set $\mathcal{E}$.

   b. In the complex plane shown below fitted with a reference frame $\left(O\,;,\overrightarrow{u}\,;\,\overrightarrow{v}\right)$ orthonormal direct, represent in red the set $\mathcal{E}$.




**E.6383** Note $\mathbb{C}$ the set of complex numbers and consider the complex plane is fitted with a reference frame $\left(O\,;\,\overrightarrow{u}\,;\,\overrightarrow{v}\right)$ orthonormé direct.




Consider the function $f$ which associates: $f(z) = z^2+2\cdot z+9$

1. Let $(F)$ be the set of points in the complex plane whose affix $z$ verifies: $\left|f(z)-8\right|=3$

   Prove that $(F)$ is the circle of center $\Omega(-1\,;0)$ and radius $\sqrt{3}$.
   Plot $(F)$ on the graph.

2. Let $z$ be a complex number, such that $z=x+\mathrm{i}\cdot y$ where $x$ and $y$ are real numbers.

   a. Show that the algebraic form of $f(z)$ is:
      $x^2-y^2+2x+9+\mathrm{i}\cdot\left(2\cdot x\cdot y+2\cdot y\right)$

   b. Let $(E)$ be the set of points in the complex plane whose affix $z$ such that $f(z)$ is a real number.
      Show that $(E)$ is the union of two straight lines $(d_1)$ and $(d_2)$ whose equations should be specified.
      Complete the graph by drawing these straight lines.

3. Determine the coordinates of the intersection points of the sets $(E)$ and $(F)$.

**E.6798** In the complex plane provided with a $(O\,;\,\overrightarrow{u}\,;\,\overrightarrow{v})$ orthonormal reference frame.

Consider the set $\mathcal{E}$ defined by: $\mathcal{E}=\{a+\mathrm{i}\cdot a \mid a\in\mathbb{R}\}$
The image of this set in the plane is the first bisector of the plane.

Consider the complex function defined on $\mathbb{C}$ by the relation:
$$f(z)=\frac{1}{z^2+1}$$
The aim of the exercise is to determine characteristics of the image of the set $\mathcal{E}'$ in the plane where:
$$\mathcal{E}'=\{f(z) \mid z\in\mathcal{E}\}$$
That is, the representation in the plane of all the images of the set $\mathcal{E}$ by the function $f$.

1. Let $z$ be an element of $\mathcal{E}$; then there exists a real $a$ such that:
$$z=a+a\cdot\mathrm{i}$$
Establish that: $f(z)=\dfrac{1}{1+4\cdot a^4}-\dfrac{2\cdot a^2}{1+4\cdot a^4}\cdot\mathrm{i}$

2. (a) Demonstrate that: $\big|f(z)-0.5\big|=0.5$
   (b) What can we say about the image of the set $\mathcal{E}'$ in the plane?
   (c) Show that the point $A$ of affix $0.5+0.5\cdot\mathrm{i}$ does not belong to the set $\mathcal{E}'$. Refine, if necessary, the answer to question (b).

## 17. Suites

**E.6381** Consider the sequence of complex numbers $(z_n)$ defined by:
$$z_0=\sqrt{3}-\mathrm{i}\quad;\quad z_{n+1}=(1+\mathrm{i})\cdot z_n$$
For any natural number $n$, we pose: $u_n=|z_n|$.

1. Calculate $u_0$.
2. Demonstrate that $(u_n)$ is the geometric sequence of reason $\sqrt{2}$ and first term 2.
3. For any natural number $n$, express $u_n$ as a function of $n$.
4. Determine the limit of the sequence $(u_n)$.

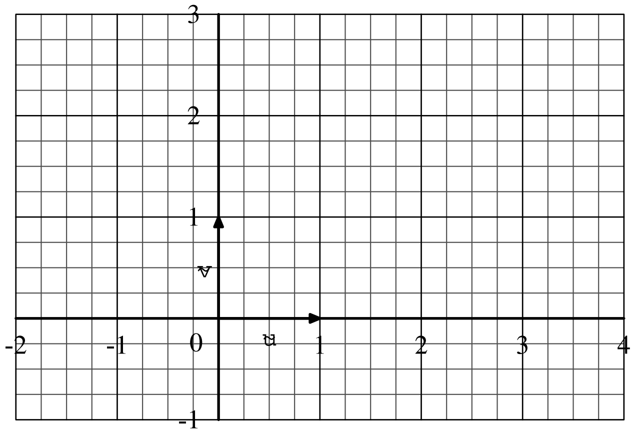
**E.6092** The complex plane is provided with a $(O\,;\,\overrightarrow{u}\,;\,\overrightarrow{v})$ orthonormal reference frame.

For any natural number $n$, let $A_n$ be the point with affix $z_n$ defined by:
$$z_0=1\quad;\quad z_{n+1}=\Big(\frac{3}{4}+\frac{\sqrt{3}}{4}\cdot\mathrm{i}\Big)\cdot z_n\quad\text{pour tout } n\in\mathbb{N}.$$
We define the sequence $(r_n)$ by $r_n=|z_n|$ for any natural number $n$.

1. Give the trigonometric form of the complex number:
$$\frac{3}{4}+\frac{\sqrt{3}}{4}\cdot\mathrm{i}.$$
2. Show that the sequence $(r_n)$ is geometric of reason $\dfrac{\sqrt{3}}{2}$
3. Determine, as a function of $n$, the measure of the segment $[A_nA_{n+1}]$.

**E.6382** Consider the sequence $(z_n)$ of complex numbers defined by: $z_0=4\quad;\quad z_{n+1}=\dfrac{1+\mathrm{i}}{2}\cdot z_n$ for all $n\in\mathbb{N}$

In the plane with a direct orthonormal reference point of origin $O$, note $A_n$ the image point of the complex number $z_n$.




1. (a) Calculate $z_1$, $z_2$ and $z_3$.
   (b) Place the points $A_1$, $A_2$ and $A_3$ in the frame below.
2. Note $\ell_n$ the length of the broken line connecting point $A_0$ to point $A_n$, passing successively through points $A_1$, $A_2$, $A_3$...

   Thus: $\ell_n=\displaystyle\sum_{k=1}^{n}A_{k-1}A_k=A_0A_1+A_1A_2+\cdots+A_{n-1}A_n$

   (a) Establish the relationship below for any non-zero natural number $n$:
$$A_{n-1}A_n=|z_n|$$
   (b) Using reasoning by recurrence, establish the following equality for any non-zero natural number $n$:
$$A_{n-1}A_n=2\sqrt{2}\cdot\Big(\frac{\sqrt{2}}{2}\Big)^{n-1}$$
   (c) Give an expression for $\ell_n$ as a function of $n$.
   (d) Determine the possible limit of the sequence $(\ell_n)$.

## 18. Unclassified financial years

**E.4314** The complex plane is referred to a $(O\,;\,\overrightarrow{u}\,;\,\overrightarrow{v})$ direct orthonormal coordinate system. We denote by $P$, $Q$ and $R$ the points with respective affixes:
$$z_P=\frac{3}{2}\cdot(1+\mathrm{i})\quad;\quad z_Q=\frac{3}{2}\cdot(1-\mathrm{i})\quad;\quad z_R=-2\cdot\mathrm{i}\cdot\sqrt{3}$$
Let $S$ be the symmetric of the point $R$ with respect to the point $Q$.

Check that the affix $z_S$ of the point $S$ is : $z_S = 3 + \mathrm{i} \cdot \left(2 \cdot \sqrt{3} - 3\right)$